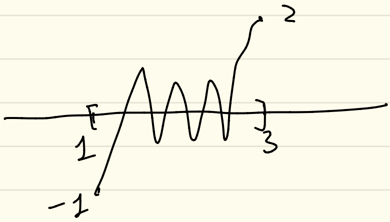
TD4:

EX1: ① Faux

② Vrai

③ Faux

④ Faux



existe $f(x)=0$
mais pas unique

⑤ Faux

eg: $f(x) = x^2 + 1$ n'a pas de racines dans $\mathbb{R}$.

TD4: Comparez avec TD3.

EX2. EX3. EX4.

① $\lim\limits_{x \to 3} x^2 + 7x = 3^2 + 21 = 30$.

EX 4.

① $\lim\limits_{n \to +\infty} \frac{3n-7}{2n+8} = \frac{3}{2}$

② $\lim\limits_{n \to +\infty} \frac{5n^3}{4n^2} = \lim \frac{5n}{4} = +\infty$

③ $\lim (\quad) = 0$

④ $\lim u_n = \frac{7}{8}$.

Ex3:

① $f(x) = \frac{2x+3}{3x^2-4}$.

$\Rightarrow \lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \frac{2x}{3x^2} = \lim\limits_{x \to +\infty} \frac{\cancel{x} \cdot 2}{\cancel{x} \cdot 3x} = 0$.

② $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \frac{2x^2+3}{3x^2-4} = \lim\limits_{x \to +\infty} \frac{2\cancel{x^2}}{3\cancel{x^2}} = \frac{2}{3}$

③ $\lim\limits_{x \to +\infty} \frac{2x^3+3}{3x^2-4} = \lim\limits_{x \to +\infty} \frac{2x^3}{3x^2} = \lim\limits_{x \to +\infty} \frac{2x}{3} = +\infty$

EX2 :

① $\lim_{x\to 3} x^2+7x = 3^2+7\times 3 = 9+21 = 30$

② $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

$(\sqrt{x}-2)(\sqrt{x}+2) = x-4$

$= \lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)}$

$= \lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)} = \lim_{x\to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$

③ $\lim_{x\to 2} \frac{x^2-2}{(x-2)^2}$.

$\lim_{x\to 2} x^2-2 = 2^2-2 = 2$

$\lim_{x\to 2} (x-2)^2 = 0$

$= +\infty$.

④ $\lim_{x\to 2} \frac{x^2-4}{(x-2)^2}$

$x^2-4 = (x-2)(x+2)$

$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x-2)} = \lim_{x\to 2} \frac{x+2}{x-2}$.

$\lim_{x\to 2^-} \frac{x+2}{x-2} = -\infty$. $\lim_{x\to 2^+} \frac{x+2}{x-2} = +\infty$. Donc, la limite n'existe pas.

⑤ $\lim_{x\to 1} \frac{x^n-1}{x-1}$. $x^n-1 = (x-1)(x^{n-1}+x^{n-2}+\dots+x+1)$.

$$= \lim_{x\to 1} (\underbrace{x^{n-1}+x^{n-2}+\dots+x+1})$$

$$= 1^{n-1}+1^{n-2}+\dots+1+1$$

$$= n.$$

⑥ $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

$\lim_{x\to 0} \sqrt{1+x}-\sqrt{1-x} = 1-1=0$

$\lim_{x\to 0} x = 0$ $\quad \frac{0}{0}$

la quantité conjuguée de $\sqrt{1+x}-\sqrt{1-x}$ est $\sqrt{1+x}+\sqrt{1-x}$

$$= \lim_{x\to 0} \frac{(\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}{x(\sqrt{1+x}+\sqrt{1-x})}$$

$$= \lim_{x\to 0} \frac{(1+x)-(1-x)}{x(\sqrt{1+x}+\sqrt{1-x})} \quad = 2x$$

$$= \lim_{x\to 0} \frac{2}{\sqrt{1+x}+\sqrt{1-x}} = \frac{2}{2} = 1.$$

EX4 :

① $\lim_{x\to +\infty} \frac{1+\sqrt{x}}{2+x}$, il s'agit d'une forme $\frac{\infty}{\infty}$

$1+\sqrt{x} = f(x)$, $2+x = g(x)$

On a $f'(x) = \frac{1}{2\sqrt{x}}$, $g'(x) = 1$.

par la règle de l'Hospital. $\lim_{x\to+\infty} \frac{1+\sqrt{x}}{2+x} = \lim_{x\to+\infty} \frac{\frac{1}{2\sqrt{x}}}{1} = 0$

② $\lim_{x\to 2} \frac{x^2-4}{x\sqrt{2x}-2x}$ il s'agit d'une forme $\frac{0}{0}$

$x^2-4 = f(x)$

$g(x) = \sqrt{2}\,x^{\frac{3}{2}}-2x$

On a $f'(x) = 2x$, $g'(x) = \sqrt{2}\cdot\frac{3}{2}\,x^{\frac{1}{2}} - 2$

$$\Rightarrow \lim_{x\to 2} \frac{2x}{\sqrt{2}\cdot\frac{3}{2}x^{\frac{1}{2}}-2} = \frac{4}{\sqrt{2}\cdot\frac{3}{2}\cdot 2^{\frac{1}{2}}-2} = \frac{4}{3-2} = 4.$$